Math 3305 Chapter 1 Section 1.1

Opening notes: Section 1.1 has 3 videos and all three scripts are in this one document. Popper 1.1 goes throughout the 3 videos and is 12 questions long. Be sure to finish and submit your answers by the deadline. There are no homework problems in 1.1; those pick up in 1.2. When you get to them, they will be turned in with all the homework problems from 1.3, 1.4 and 1.5 in one document under "Homework Chapter 1" under the assignment tab in CourseWare. There are two essays (One, page 16 and Two, page 34) will be turned in individually also under the assignments tab; one per assignment; the essays are discussed in the videos and are written out in this document. Be sure to stick to one page. PDF them and turn them in by the deadline. Deadlines are in the Course Calendar. NOW!

Video 1 for 1.1. The Ancient Stuff and some history.

We begin with the STRUCTURE of geometry. A geometry is an axiomatic system. Note "A" not "The".

Some other geometries in additional to traditional Euclidean Geometry:

Big Ones: Spherical, Hyperbolic, Finsler, …

Little Ones: Triangle, Klein, Fano's,…

Areas of study other than Geometry have an axiomatic structure. Probability and Logic for example, and Real Analysis. We will restrict ourselves to Geometry and it's axiomatic structure.

Modern axiomatic system parts:

Undefined terms — shortest possible list — point, line, on, space, half plane

Axioms — the rules try for short list

Definitions — #3 on the list

Theorems – lemmas and corollaries — sequel / prequel — short proofs

The first axiomatic system was devised by Euclid and he recorded it in his book Elements centuries before Christ was born. It was WAY shorter than what we use now. Let's look at it briefly and I'll point out some problems that have been noticed and fixed over the centuries since his time.

Euclid's system:

Common Notions

Definitions — we have these just not in the intro

Postulate — now "axiom"

Theorems

Definitions — more

He began with Common Notions. These are now reduced to vocabulary and vocabulary. We no longer list the information BEHIND the system. Most of those have their own axiomatic systems now. → arith. & logic, number theory

CN1 Things that are equal to the same thing are also equal to each other.

$A = B \wedge B = C \therefore A = C$

The transitive property of equality – Equivalence Relations in 2.1

CN2 If equals are added to equals, the sums are equal. $A + B = C + B$

CN3 If equals are subtracted from equals, the remainders are equal.

$A - B = C - B$

CN4 Things that coincide with each other are equal to each other.

equal now means same point sets

We use Congruent

length, angle measure, area

CN5 The whole is greater than the part. Not so any more

$-5 = -3 + -2$

These are behind-the-scenes facts. We no longer list them. They are huge systems in their own right now

He then went on to 26 definitions. Let's look at a couple:

A point is that which has no part.

A line is a breadthless length.

The ends of a line are points.

A straight line is a line which lies evenly with the points on itself.

A surface is that which has length and breadth only.

The edges of a surface are lines.

We don't do this any more. The undefined terms in the axiomatic system I'll teach you in the next video are: point, line, half-plane, and plane. We will have MANY definitions but every geometry starts with undefined terms now. And fewer is considered better now.

He then went on to his Postulates (now called axioms).

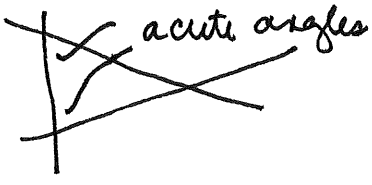
P1 You may draw a straight line from any point to any point.

P2 You may draw a straight line continuously from one line to another.

P3 You may draw a circle with any center and radius.

P4 Know that each right angle is equal to each other right angle.

P5 If a straight line crossing two other straight line has angles on one side less that right angles, then those two lines crossed meet on that side somewhere.




restatement

In the next video: 22 axioms. In Birkhoff: 5 axioms. In middle school books sometimes 40 or 50 axioms!

**Popper 1.1, Question One**

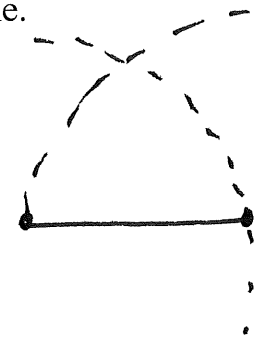
Euclid began what is now known as an Axiomatic System centuries ago.

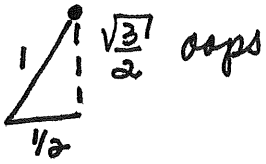
A. True

B. False

Then Theorems – 13 books of theorems! Let's just look at the first two. These are constructions. Done occasionally now. We have rulers, though, and protractors. Euclid didn't have those. And he only had natural numbers!

Theorem 1.1 You may construct an equilateral triangle on a given straight line.






Theorem 1.2 You may place a straight line equal to a given straight line on the end of another given line.

straight edge & compass

now we have ∞

This finishes up the ancient part of our look at Geometry. Many people like to hark back to Euclid's axioms when talking geometry but I think it's MUCH better to go with modern axioms and be totally complete with our ideas and statements.

★ Video 2 for Chapter 1 Section 1.

Mostly new, more modern information. Popper 1.1 continues.

From 1958 – 1977 the Schoolhouse Mathematics Study Group met regularly to work on producing a study guide for a national curriculum at the high school level for Euclidean Geometry. They published the following in 1961.

The SMSG Axioms for Euclidean Geometry

Undefined terms
Axioms

Theorems and Definitions came along later.

SMSG Undefined Terms for Euclidean Geometry:

point, line, half plane, plane, and space

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We take these as our beginning point. We can visualize, sketch, or model, just not define.

Most people visualize a point as a tiny, tiny dot. Lines are thought of as long, seamless concatenations of points and planes are composed of finely interwoven lines: smooth, endless and flat.

Think of undefined terms as the basic sounds in a language – the sounds that make up our language for the most part have no meaning in themselves but are combined to make words.

The grammar of our language and a good dictionary are what make the meaning of the sounds. This part of language corresponds to the axioms and definitions that you will find as we move along in the sections and chapters.